GROUP ON EARTH OBSERVATIONS

QA4EO A QUALITY ASSURANCE FRAMEWORK FOR EARTH OBSERVATION

A guide to expression of uncertainty of measurements

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A guide to expression of uncertainty of measurements

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Version 3.0 now includes edits and comments made on version 2.0 by GSICS in March 2009.

Version 4.0 incorporates edits and comments made on version 3.0 by WMO and GSICS August – November 2009.

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1 Abstract

This key guideline provides an introduction to the ISO 'Guide to the Expression of Uncertainty in Measurement' (GUM) [1]. Confidence in a measured value requires a quantitative statement of its quality, which in turn necessitates the evaluation of the uncertainty associated with the value. The basis for the value and the associated uncertainty is "traceability", involving the relationship of relevant quantities to national or international (community agreed) standards through an unbroken chain of comparisons. Each comparison involves calibration of a standard at one level in the chain using a standard at a higher level. The concept must involve all relevant processes and be fully transparent. This principle is common to all scientific activities and Earth Observation (EO) is no exception. QA4EO is built on this principle with uncertainty at its core. The evaluation of uncertainty of measurement is founded on the use of models of measurement for each stage of the chain, which are detailed in the GUM. This document provides a brief introduction to the GUM and is itself based on other more generic published texts [2, 3].

2 Scope

A key requirement driving the data quality assurance aspects of the Group on Earth Observations (GEO)'s Quality Assurance Framework for Earth Observation (QA4EO) is the need for interoperability and the means to allow all stakeholders to be able to readily assess (on receipt) the suitability of a data (or derived) product for their particular application. This requires a "quality indicator" (QI), which must be unequivocal and universal in terms of its definition and derivation. In practise there is likely to be a wide range of actual descriptors and terms used (e.g., text or numeric) depending on the specific application or user needs, but all should be based on a statistically derived value. This value should be the result of an assessment of its traceability to a community agreed "reference standard" (ideally SI) as propagated through the data processing chain.

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The 'Guide to the Expression of Uncertainty in Measurement' (GUM) [1] provides general rules for evaluating and expressing uncertainty in measurement that are intended to be applicable to a wide range of measurements and for use within standardisation, calibration, laboratory accreditation and measurement services. The basis of the GUM is Recommendation INC–1 (1980), 'Expression of experimental uncertainties' [4], of the Working Group on the Statement of Uncertainties, convened in 1980 by the Bureau International des Poids et Mesures (BIPM) in response to a request by the Comitée International des Poids et Mesures (CIPM). The CIPM approved the Recommendation in 1981 [5] and reaffirmed it in 1986 [6]. The responsibility for developing a detailed guide based on the Working Group's recommendation was given to the Technical Advisory Group on Metrology (TAG4) of the International Organization for Standardization (ISO), in which six other international organisations were represented, namely the BIPM, the International Electrotechnical Commission (IEC), the International Federation of Clinical Chemistry and Laboratory Medicine (IFCC), the International Union of Pure and Applied Chemistry (IUPAC), the International Union of Pure and Applied Physics (IUPAP) and the International Organisation of Legal Metrology (OIML). The resulting document was published in 1993 and reprinted with minor corrections in 1995 [1]. It is this latest guide that is recommended for use by the EO community.

Many supplementary guides are being drafted to supplement this guide, including specific guidance on models as well as more introductory texts. These will become available from the BIPM website in due course: http://www.bipm.org/.

Although highlighting the principles of uncertainty evaluation, this document leaves all the detail to the GUM and forthcoming supplements.

3 Terminology

All terms within this document are based on internationally-agreed definitions that are, in many cases, derived directly from formal standardising bodies such as ISO. These agreed definitions can be found on the QA4EO website (http://QA4EO.org/).

4 Introduction / Context

This key guideline is written as part of a set, based on the adoption of existing best practise, to form a Quality Assurance Framework for Earth Observation (QA4EO). The QA4EO was developed to meet the current and aspirational needs of the societal themes of the Group on Earth Observation (GEO)'s Global Earth Observation System of Systems (GEOSS). It was prepared as a direct response to GEO task DA-06-02 (now DA-09-01-a) to "Develop a GEO data quality assurance strategy, beginning with space-based observations and evaluating expansion to *in situ* observations, taking account of existing work in this arena".

5 Outcomes

The outcome of this guideline is a link to the ISO guide on 'Expression of Uncertainty in Measurement'; the GUM [1]. Following the guidance contained within the GUM will provide the reader with the tools to evaluate and express uncertainty in their measurement result or process.

6 Inputs

The key inputs that are needed when using the GUM [1] will be a detailed analysis and understanding of the measurement or process to allow a "model" and interdependencies to be established.

7 Standards and Traceability

This document provides the link to the GUM [1], which is the recognised standard.

8 Evaluation and expression of uncertainty

8.1 Introduction

Users of data products require confidence in those products. The products are the result of measurements and measurement-related processes. Confidence is obtained from a quantitative statement of the quality of the results, i.e. the Quality Indicator (QI). The statement is built upon an evaluation of the uncertainties associated with the overall process. In itself such information is insufficient; it is necessary to have in place a credible basis for the measurement results and the associated uncertainties. The basis constitutes traceability of the measurement results through an unbroken chain of comparisons to stated references, typically in the form of standards held by National Measurement Institutes (NMIs) or designated laboratories, or as defined by the international community.

8.2 Uncertainty Evaluation

Uncertainty evaluation involves the use of a model of measurement to determine the uncertainty. Associated with this is a best estimate of the value of the quantity to be measured, given best estimates of the values of all quantities that significantly influence that quantity and prescribed uncertainties associated with those estimates. The 'Guide to the Expression of Uncertainty of Measurement' (GUM) [1] and several documents derived from it provide guidance for a wide range of uncertainty evaluation problems. Supplements to the GUM are being prepared by the Joint Committee for Guides in Metrology [7]. These supplements aim to give added value to the GUM by enlarging the set of problems that can validly be addressed.

The main stages of uncertainty evaluation can be described as follows:

i. Define the output quantity Y, the quantity to be measured;

ii. Decide the input quantities $\boldsymbol{X} = (X1, \ldots, XN)$T upon which the output quantity depends;

iii. Develop a model $Y = f(\boldsymbol{X})$ relating the output quantity to these input quantities;

iv. On the basis of available knowledge assign probability density functions (PDFs) (GUM clause C.2.5)—Gaussian (normal), rectangular (uniform), etc.—to the values of the Xi ;

v. *Propagate* the PDFs for the values of the *Xi* through the model to obtain the PDF for the value of *Y*;

vi. Use the PDF for the value of *Y* to obtain

a. the expectation of that value, taken as the best estimate y of the value of Y;

b. the standard deviation of that value, taken as the standard uncertainty u(y) associated with y (GUM clause E.3.2);

c. an interval (the coverage interval) containing the value of Y with a specified high probability (the coverage probability).

Stages (i)–(iv) are regarded as *formulation* and stages (v) and (vi) as *calculation*. The former stages are carried out by the technical expert. The latter stages require no further metrological information and, in principle, can be carried out to any required degree of numerical approximation, relative to the problem specified at the formulation stage.

8.3 GUM uncertainty framework

The GUM provides general guidance on many aspects of the above stages. It also provides the *GUM uncertainty framework* for the calculation phase of uncertainty evaluation.

The GUM uncertainty framework has been adopted by many organisations, is widely used and has been implemented in standards and guides on measurement uncertainty and in computer packages. To apply the framework, the values of the model input quantities *Xi* are summarised by their expectations and standard deviations, e.g., as given by the PDFs for these values (GUM clause 4.1.6). This information is "propagated" through a first- or higher-order Taylor series approximation to the model to provide a best estimate *y* of the value of the output quantity *Y* and the associated standard uncertainty *u(y)*. The best estimate is given by evaluating the model at the best estimate $\boldsymbol{x}$ of the vector input quantity value $\boldsymbol{X}$. A coverage interval for the value of *Y* is provided based on taking the PDF for the output quantity value as Gaussian or, if the (effective) degrees of freedom associated with the standard deviation of the PDF for the value of *Y* is finite (GUM clause G), as a scaled and shifted *t*-distribution.

Specifically, the calculation stages (stages (v) and (vi)) of the GUM uncertainty framework can be summarised as:

i. Obtain from the PDFs for the values of the input quantities $X_1, \ldots, X_N$ the expectations $\boldsymbol{x} = (x_1, \ldots, x_N)^T$ and the standard deviations (standard uncertainties) $\boldsymbol{u(x)} = (u(x_1), \ldots, u(x_N))^T$;

ii. For each pair i, j for which the values of X_i and X_j are mutually dependent, obtain from the joint PDF for the values of X_i and X_j the *covariance* (mutual uncertainty) (GUM clause C) $u(x_i, x_j)$ associated with x_i and x_j;

iii. Form the partial derivatives of first order of f with respect to the input quantities;

iv. Calculate the best estimate y of the output quantity value by evaluating the model at the fixed value of $\boldsymbol{X}$ equal to $\boldsymbol{x}$;

v. Calculate the model sensitivity coefficients (GUM clause 5.1) as the above partial derivatives evaluated at $\boldsymbol{x}$;

vi. Determine the standard uncertainty $u(y)$ by combining $\boldsymbol{u(x)}$, the $u(x_i, x_j)$ and the model sensitivity coefficients (GUM formulae (10) and (13));

vii. Calculate ν, the effective degrees of freedom associated with $u(y)$, using the Welch–Satterthwaite formula (GUM formula (G.2b));

viii. Compute the expanded uncertainty U_p, and hence a coverage interval (for a stipulated coverage probability p), for the output quantity value η of Y, regarded as a random variable, by forming the appropriate multiple of $u(y)$ through taking the PDF of $(\eta - y)/u(y)$ as a standard Gaussian distribution $(\nu = \infty)$ or t–distribution $(\nu < \infty)$.

8.4 References

1. BIPM, IEC, IFCC, ISO, IUPAC, IUPAP and OIML (1995) Guide to the Expression of Uncertainty in Measurement 2nd edn ISBN 92-67-10188-9.

2. Cox M G, Harris P. M. *Meas. Sci. Technol.* **17** (2006) 533–540.

3. Birch W., Cox, M. G., Harris P. M. *Metrologia* **43** (2006) S161–S166.

4. Kaarls R *BIPM Proc.-Verb. Com. Int. Poids et Mesures* **49** (1981) A1–12 (in French). Giacomo P *Metrologia* **17** (1981) 73–4 (in English).

5. CIPM *BIPM Proc.-Verb. Com. Int. Poids et Mesures* **49** (1981) 8–9, 26 (in French). Giacomo P 1982 *Metrologia* **18** 43–4 (in English).

6. CIPM *BIPM Proc.-Verb. Com. Int. Poids et Mesures* **54** (1986)14, 35 (in French). Giacomo P *Metrologia* **24** (1987) 49–50 (in English).

7. http://www.bipm.org/en/committees/jc/jcgm.

9 Conclusion

This document provides a brief introduction to the ISO guide to the 'Guide to the Expression of Uncertainty of Measurement' (GUM) [1] . The GUM provides the tools and examples to allow readers to evaluate and document the uncertainty of their measurement-based process in a rigorous and harmonised manner. Its use is fundamental to the success of QA4EO and additional EO specific guidance may be developed to aid the community in its implementation.